

Exercise 33

For the following exercises, find the average rate of change of each function on the interval specified.

$$p(t) = \frac{(t^2 - 4)(t + 1)}{t^2 + 3} \text{ on } [-3, 1]$$

Solution

The average rate of change of the function between $t = -3$ and $t = 1$ is

$$\begin{aligned} \frac{p(1) - p(-3)}{1 - (-3)} &= \frac{\frac{[(1)^2 - 4][(1) + 1]}{(1)^2 + 3} - \frac{[(-3)^2 - 4][(-3) + 1]}{(-3)^2 + 3}}{1 + 3}} \\ &= \frac{\frac{(1-4)(1+1)}{1+3} - \frac{(9-4)(-3+1)}{9+3}}{4} \\ &= \frac{\frac{(-3)(2)}{4} - \frac{(5)(-2)}{12}}{4} \\ &= \frac{\frac{-6}{4} - \frac{-10}{12}}{4} \\ &= \frac{\frac{-6(3)}{4(3)} - \frac{-10}{12}}{4} \\ &= \frac{\frac{-18}{12} - \frac{-10}{12}}{4} \\ &= \frac{\frac{-18 - (-10)}{12}}{4} \\ &= \frac{\frac{-18 + 10}{12}}{4} \\ &= \frac{\frac{-8}{12}}{4} \\ &= \frac{-8}{4(12)} \\ &= -\frac{2}{12} \\ &= -\frac{1}{6}. \end{aligned}$$